Exclusive weak B decays to excited meson states

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Exclusive semileptonic B decays to radially excited charmed mesons and rare radiative B decay to the orbitally excited tensor $K_2^*(1430)$ meson are investigated in the framework of the relativistic quark model based on the quasipotential approach in quantum field theory. The heavy quark expansion is applied for the semileptonic $B \to D^{(*)\prime} e\nu$ decays. Both relativistic and the $1/m_Q$ corrections are found to play an important role for these decays and substantially modify results. For rare radiative $B \to K_2^*(1430)\gamma$ decay such an expansion is not used for the s quark. Instead we apply the expansion in inverse powers of large recoil momentum of final $K_2^*(1430)$ meson. The calculated branching fraction $BR(B \to K_2^*(1430)\gamma) = (1.7 \pm 0.6) \times 10^{-5}$ as well as the ratio $BR(B \to K_2^*(1430)\gamma)/BR(B \to K^*(892)\gamma) = 0.38 \pm 0.08$ are found to be in a good agreement with recent experimental data from CLEO.

I. INTRODUCTION

The investigation of weak B decays to excited mesons presents a problem interesting both from the experimental and theoretical point of view. The current experimental data on semileptonic B decays to ground state D mesons indicate that a substantial part ($\approx 40\%$) of the inclusive semileptonic B decays should go to excited D meson states. First experimental data on some exclusive B decay channels to excited charmed mesons are becoming available now [1, 2, 3] and more data are expected in near future. Thus the comprehensive theoretical study of these decays is necessary. The presence of a heavy quark in the initial and final meson states in these decays considerably simplifies their theoretical description. A good

starting point for this analysis is the infinitely heavy quark limit, $m_Q \to \infty$ [4]. In this limit the heavy quark symmetry arises, which strongly reduces the number of independent weak form factors [5]. The heavy quark mass and spin then decouple and all meson properties are determined by light-quark degrees of freedom alone. This leads to a considerable reduction of the number of independent form factors which are necessary for the description of heavy-to-heavy semileptonic decays. For example, in this limit only one form factor is necessary for the semileptonic B decay to S-wave D mesons (both for the ground state and its radial excitations), while the decays to P states require two form factors [5]. It is important to note that in the infinitely heavy quark limit matrix elements between a B meson and an excited D meson should vanish at the point of zero recoil of the final excited charmed meson in the rest frame of the B meson. In the case of B decays to radially excited charmed mesons this follows from the orthogonality of radial parts of wave functions, while for the decays to orbital excitations this is the consequence of orthogonality of their angular parts. However, some of the $1/m_Q$ corrections to these decay matrix elements give nonzero contributions at zero recoil. As a result the role of these corrections could be considerably enhanced, since the kinematical range for B decays to excited states is a rather small region around zero recoil.

Rare radiative decays of B mesons are induced by flavour changing neutral currents and thus they can serve as sensitive probes of new physics beyond the standard model. Such decays are governed by one-loop (penguin) diagrams with the main contribution from a virtual top quark and a W boson. The statistics of rare radiative B decays considerably increased since the first observation of the $B \to K^*\gamma$ decay in 1993 by CLEO [6]. This allowed a significantly more precise determination of exclusive and inclusive branching fractions [7]. Recently the first observation of rare B decay to the orbitally excited tensor strange meson $B \to K_2^*(1430)\gamma$ has been reported by CLEO [7] with a branching fraction

$$BR(B \to K_2^*(1430)\gamma) = (1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5}, \tag{1}$$

as well as the ratio of exclusive branching fractions

$$r \equiv \frac{BR(B \to K_2^*(1430)\gamma)}{BR(B \to K^*(892)\gamma)} = 0.39^{+0.15}_{-0.13}. \tag{2}$$

These new experimental data provide a challenge to the theory. Many theoretical approaches have been employed to predict the exclusive $B \to K^*(892)\gamma$ decay rate (for a review see

[8] and references therein). Less attention has been payed to rare radiative B decays to excited strange mesons [9, 10, 11]. Most of these theoretical approaches [10, 11] rely on the heavy quark limit both for the initial b and final s quark and the nonrelativistic quark model. However, the two predictions [10, 11] for the ratio r in Eq. (2) differ by an order of magnitude, due to a different treatment of the long distance effects and, as a result, a different determination of corresponding Isgur-Wise functions. Only the prediction of Ref. [11] is consistent with data (1), (2). Nevertheless, it is necessary to point out that the s quark in the final K^* meson is not heavy enough, compared to the $\bar{\Lambda}$ parameter, which determines the scale of $1/m_Q$ corrections in heavy quark effective theory [12]. Thus the $1/m_s$ expansion is not appropriate. Notwithstanding, the ideas of heavy quark expansion can be applied to the exclusive $B \to K^*(K_2^*)\gamma$ decays. From the kinematical analysis it follows that the final $K^*(K_2^*)$ meson bears a large relativistic recoil momentum $|\mathbf{\Delta}|$ of order of $m_b/2$ and an energy of the same order. So it is possible to expand the matrix element of the effective Hamiltonian both in inverse powers of the b quark mass for the initial state and in inverse powers of the recoil momentum $|\mathbf{\Delta}|$ for the final state [13, 14].

Our relativistic quark model is based on the quasipotential approach in quantum field theory with a specific choice of the quark-antiquark interaction potential. It provides a consistent scheme for the calculation of all relativistic corrections at a given v^2/c^2 order and allows for the heavy quark $1/m_Q$ expansion. In preceding papers we applied this model to the calculation of the mass spectra of orbitally and radially excited states of heavy-light mesons [15], as well as to the description of the rare radiative decay $B \to K^*\gamma$ [13] and of weak decays of B mesons to ground state heavy and light mesons [16, 17]. The heavy quark expansion for the ground state heavy-to-heavy semileptonic transitions [18] was found to be in agreement with model-independent predictions of the heavy quark effective theory (HQET).

II. RELATIVISTIC QUARK MODEL

In our model a meson is described by the wave function of the bound quark-antiquark state, which satisfies the quasipotential equation [19] of the Schrödinger type [20]:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q}), \tag{3}$$

where the relativistic reduced mass is

$$\mu_R = \frac{M^4 - (m_q^2 - m_Q^2)^2}{4M^3}. \tag{4}$$

In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_q + m_Q)^2][M^2 - (m_q - m_Q)^2]}{4M^2}. \tag{5}$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (3) is the quasipotential operator of the quark-antiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive energy states. An important role in this construction is played by the Lorentz-structure of the confining quark-antiquark interaction in the meson. In constructing the quasipotential of the quark-antiquark interaction we have assumed that the effective interaction is the sum of the usual one-gluon exchange term and the mixture of vector and scalar linear confining potentials. The quasipotential is then defined by [21]

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) &= \bar{u}_q(p)\bar{u}_Q(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_q(q)u_Q(-q) \\ &= \bar{u}_q(p)\bar{u}_Q(-p)\left\{\frac{4}{3}\alpha_s D_{\mu\nu}(\mathbf{k})\gamma_q^\mu\gamma_Q^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_q^\mu\Gamma_{Q;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_q(q)u_Q(-q), \end{aligned} \tag{6}$$

where α_s is the QCD coupling constant, $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p) + m} \end{pmatrix} \chi^\lambda \tag{7}$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$. The effective long-range vector vertex is given by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu, \tag{8}$$

where κ is the Pauli interaction constant characterizing the nonperturbative anomalous chromomagnetic moment of quarks. Vector and scalar confining potentials in the nonrelativistic limit reduce to

$$V^V_{\rm conf}(r) = (1 - \varepsilon)(Ar + B), \qquad V^S_{\rm conf}(r) = \varepsilon(Ar + B), \tag{9}$$

reproducing

$$V_{\rm conf}(r) = V^S_{\rm conf}(r) + V^V_{\rm conf}(r) = Ar + B, \tag{10}$$

where ε is the mixing coefficient.

The quasipotential for the heavy quarkonia, expanded in v^2/c^2, can be found in Refs. [21, 22] and for heavy-light mesons in [15]. All the parameters of our model, such as quark masses, parameters of the linear confining potential, mixing coefficient ε and anomalous chromomagnetic quark moment κ, were fixed from the analysis of heavy quarkonia masses [21] and radiative decays [23]. The quark masses $m_b = 4.88$ GeV, $m_c = 1.55$ GeV, $m_s = 0.50$ GeV, $m_{u,d} = 0.33$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.30$ GeV have the usual quark model values. In Ref. [18] we have considered the expansion of the matrix elements of weak heavy quark currents between pseudoscalar and vector meson ground states up to the second order in inverse powers of the heavy quark masses. It has been found that the general structure of the leading, first, and second order $1/m_Q$ corrections in our relativistic model is in accord with the predictions of HQET. The heavy quark symmetry and QCD impose rigid constraints on the parameters of the long-range potential in our model. The analysis of the first order corrections [18] fixes the value of the Pauli interaction constant $\kappa = -1$. The same value of κ was found previously from the fine splitting of heavy quarkonia ^3P_J- states [21]. The value of the parameter mixing vector and scalar confining potentials $\varepsilon = -1$ was found from the analysis of the second order corrections [18]. This value is very close to the one determined from considering radiative decays of heavy quarkonia [23].

In the quasipotential approach, the matrix element of the weak current $J_\mu = \bar{f}Gb$ ($f = \{c$ or $s\}$, $G = \{\gamma^\mu(1-\gamma^5)$ or $\frac{i}{2}k^\nu\sigma_{\mu\nu}(1+\gamma^5)\}$) between the states of a B meson and an excited F ($D^{(*)\prime}$ or K_2^*) meson has the form [24]

$$\langle F|J_\mu(0)|B\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6}\bar{\Psi}_F(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_B(\mathbf{q}), \tag{11}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{B,F}$ are the meson wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame. The contributions to Γ come from Figs. 1 and 2. The contribution $\Gamma^{(2)}$ is the consequence of the projection onto the positive-energy states. Note that the form of the relativistic corrections resulting from the vertex function $\Gamma^{(2)}$ explicitly depends on the Lorentz structure of the $q\bar{q}$-interaction. The vertex functions look like

$$\Gamma_\mu^{(1)}(\mathbf{p},\mathbf{q}) = \bar{u}_f(p_1)Gu_b(q_1)(2\pi)^3\delta(\mathbf{p}_2-\mathbf{q}_2), \tag{12}$$

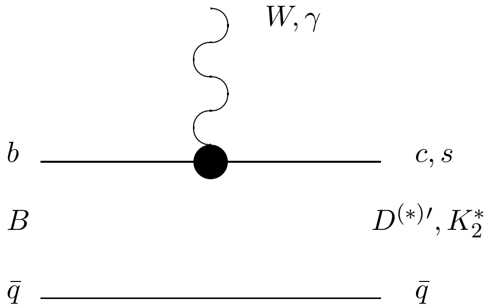

FIG. 1. Lowest order vertex function $\Gamma^{(1)}$ corresponding to Eq. (12).

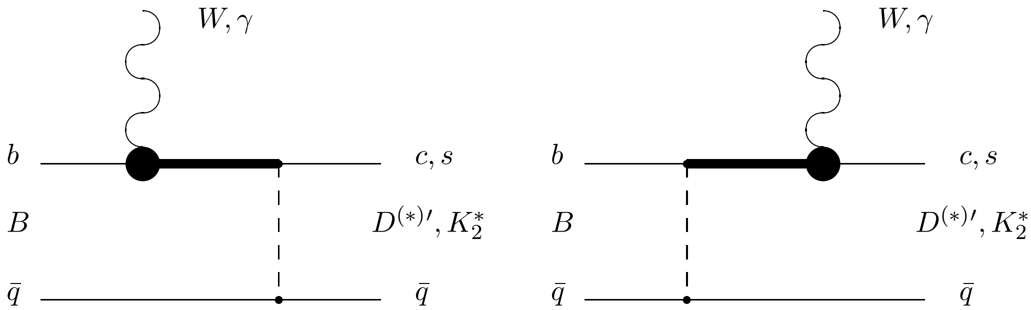

FIG. 2. Vertex function $\Gamma^{(2)}$ corresponding to Eq. (13). Dashed lines represent the effective potential $\mathcal{V}$ in Eq. (6). Bold lines denote the negative-energy part of the quark propagator.

and

$$\Gamma_\mu^{(2)}(\mathbf{p},\mathbf{q}) = \bar u_f(p_1)\bar u_q(p_2)\Big\{G\frac{\Lambda_b^{(-)}(k_1)}{\epsilon_b(k_1)+\epsilon_b(p_1)}\gamma_1^0\mathcal{V}(\mathbf{p}_2-\mathbf{q}_2) + \mathcal{V}(\mathbf{p}_2-\mathbf{q}_2)\frac{\Lambda_f^{(-)}(k_1')}{\epsilon_f(k_1')+\epsilon_f(q_1)}\gamma_1^0 G\Big\}u_b(q_1)u_q(q_2), \tag{13}$$

where $\mathbf{k}_1=\mathbf{p}_1-\mathbf{\Delta}$; $\quad \mathbf{k}_1'=\mathbf{q}_1+\mathbf{\Delta}$; $\quad \mathbf{\Delta}=\mathbf{p}_F-\mathbf{p}_B$;

$$\Lambda^{(-)}(p)=\frac{\epsilon(p)-(m\gamma^0+\gamma^0(\boldsymbol{\gamma}\mathbf{p}))}{2\epsilon(p)}.$$

It is important to note that the wave functions entering the weak current matrix element (11) cannot be both in the rest frame. In the B meson rest frame, the F meson is moving with the recoil momentum $\mathbf{\Delta}$. The wave function of the moving F meson $\Psi_{F\,\mathbf{\Delta}}$ is connected with the F wave function in the rest frame $\Psi_{F\,\mathbf{0}}\equiv\Psi_F$ by the transformation [24]

$$\Psi_{F\,\mathbf{\Delta}}(\mathbf{p})=D_f^{1/2}(R_{L_{\mathbf{\Delta}}}^W)D_q^{1/2}(R_{L_{\mathbf{\Delta}}}^W)\Psi_{F\,\mathbf{0}}(\mathbf{p}), \tag{14}$$

where R^W is the Wigner rotation, $L_{\boldsymbol{\Delta}}$ is the Lorentz boost from the meson rest frame to a moving one. The wave functions of B, $D^{(*)\prime}$ and K_2^* mesons at rest were calculated by numerical solution of the quasipotential equation (3).

III. SEMILEPTONIC DECAYS TO RADIALLY EXCITED STATES

The matrix elements of the vector and axial vector currents between the B and radially excited D' or $D^{*\prime}$ mesons can be parameterized by six hadronic form factors:

$$\begin{aligned}
\frac{\langle D'(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_{D'}m_B}} &= h_+(v+v')^\mu + h_-(v-v')^\mu,\\
\langle D'(v')|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle &= 0,\\
\frac{\langle D^{*\prime}(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_{D^{*\prime}}m_B}} &= ih_V\varepsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha v'_\beta v_\gamma,\\
\frac{\langle D^{*\prime}(v',\epsilon)|\bar{c}\gamma^\mu\gamma_5 b|B(v)\rangle}{\sqrt{m_{D^{*\prime}}m_B}} &= h_{A_1}(w+1)\epsilon^{*\mu} - (h_{A_2}v^\mu + h_{A_3}v'^\mu)(\epsilon^*\cdot v), \qquad (15)
\end{aligned}$$

where v (v') is the four-velocity of the B ($D^{(*)\prime}$) meson, ϵ^μ is a polarization vector of the final vector charmed meson, and the form factors h_i are dimensionless functions of the product of velocities $w = v\cdot v'$.

The HQET analysis [25] shows that five independent functions $\tilde{\xi}_3$, χ_b and $\tilde{\chi}_{1,2,3}$, as well as two mass parameters $\bar{\Lambda}$ and $\bar{\Lambda}^{(n)}$ are necessary to describe first order $1/m_Q$ corrections to matrix elements of B meson decays to radially excited D meson states. The function $\tilde{\xi}_3$ emerges from corrections to the current in effective theory, while χ_b and $\tilde{\chi}_{1,2,3}$ parameterize corrections to HQET Lagrangian. The resulting structure of the decay form factors is [25]

$$\begin{aligned}
h_+ &= \xi^{(n)} + \varepsilon_c\left[2\tilde{\chi}_1 - 4(w-1)\tilde{\chi}_2 + 12\tilde{\chi}_3\right] + \varepsilon_b\chi_b,\\
h_- &= \varepsilon_c\left[2\tilde{\xi}_3 - \left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}\right] - \varepsilon_b\left[2\tilde{\xi}_3 - \left(\bar{\Lambda} - \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}\right],\\
h_V &= \xi^{(n)} + \varepsilon_c\left[2\tilde{\chi}_1 + \left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)} - 4\tilde{\chi}_3\right] + \varepsilon_b\left[\chi_b + \left(\bar{\Lambda} - \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)} - 2\tilde{\xi}_3\right],\\
h_{A_1} &= \xi^{(n)} + \varepsilon_c\left[2\tilde{\chi}_1 - 4\tilde{\chi}_3 + \frac{w-1}{w+1}\left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)}\right]\\
&\quad + \varepsilon_b\left\{\chi_b + \frac{w-1}{w+1}\left[\left(\bar{\Lambda} - \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)} - 2\tilde{\xi}_3\right]\right\},\\
h_{A_2} &= \varepsilon_c\left\{4\tilde{\chi}_2 - \frac{2}{w+1}\left[\left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)} + \tilde{\xi}_3\right]\right\},\\
h_{A_3} &= \xi^{(n)} + \varepsilon_c\left[2\tilde{\chi}_1 - 4\tilde{\chi}_2 - 4\tilde{\chi}_3 + \frac{w-1}{w+1}\left(\bar{\Lambda}^{(n)} + \frac{\bar{\Lambda}^{(n)}-\bar{\Lambda}}{w-1}\right)\xi^{(n)} - \frac{2}{w+1}\tilde{\xi}_3\right]
\end{aligned}$$

$$+\varepsilon_b\left[\chi_b+\left(\bar\Lambda-\frac{\bar\Lambda^{(n)}-\bar\Lambda}{w-1}\right)\xi^{(n)}-2\tilde\xi_3\right], \qquad (16)$$

where $\varepsilon_Q = 1/(2m_Q)$ and $\bar\Lambda(\bar\Lambda^{(n)}) = M(M^n) - m_Q$.

Now we can perform the heavy quark expansion for the matrix elements of B decays to excited D mesons in the framework of our model and determine leading and subleading Isgur–Wise functions. To do this we substitute the vertex functions $\Gamma^{(1)}$ and $\Gamma^{(2)}$ given by Eqs. (12) and (13) in the decay matrix element (11) and take into account the wave function properties (14). The resulting structure of this matrix element is rather complicated, because it is necessary to integrate both over d^3p and d^3q. The δ function in expression (12) permits us to perform one of these integrations and thus this contribution can be easily calculated. The calculation of the vertex function $\Gamma^{(2)}$ contribution is more difficult. Here, instead of a δ function, we have a complicated structure, containing the $Q\bar q$ interaction potential in the meson. However, we can expand this contribution in inverse powers of heavy (b, c) quark masses and then use the quasipotential equation in order to perform one of the integrations in the current matrix element. We carry out the heavy quark expansion up to first order in $1/m_Q$. It is easy to see that the vertex function $\Gamma^{(2)}$ contributes already at the subleading order of the $1/m_Q$ expansion. Then we compare the arising decay matrix elements with the form factor decompositions (16) for decays to radial excitations and determine the form factors. We find that, for the chosen values of our model parameters (the mixing coefficient of vector and scalar confining potential $\varepsilon = -1$ and the Pauli constant $\kappa = -1$), the resulting structure at leading and subleading order in $1/m_Q$ coincides with the model-independent predictions of HQET. This allows us to determine leading and subleading Isgur-Wise functions [25]:

$$\xi^{(1)}(w) = \left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar\psi^{(0)}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \qquad (17)$$

$$\tilde\xi_3(w) = \left(\frac{\bar\Lambda^{(1)}+\bar\Lambda}{2}-m_q+\frac{1}{6}\frac{\bar\Lambda^{(1)}-\bar\Lambda}{w-1}\right)\left(1+\frac{2}{3}\frac{w-1}{w+1}\right)\xi^{(1)}(w), \qquad (18)$$

$$\begin{aligned}\tilde\chi_1(w) \cong\ & \frac{1}{20}\frac{w-1}{w+1}\frac{\bar\Lambda^{(1)}-\bar\Lambda}{w-1}\xi^{(1)}(w)\\ &+\frac{\bar\Lambda^{(1)}}{2}\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar\psi^{(1)si}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \qquad (19)\end{aligned}$$

$$\tilde\chi_2(w) \cong -\frac{1}{12}\frac{1}{w+1}\frac{\bar\Lambda^{(1)}-\bar\Lambda}{w-1}\xi^{(1)}(w), \qquad (20)$$

$$\tilde\chi_3(w) \cong -\frac{3}{80}\frac{w-1}{w+1}\frac{\bar\Lambda^{(1)}-\bar\Lambda}{w-1}\xi^{(1)}(w)$$

$$+\frac{\bar\Lambda^{(1)}}{4}\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar\psi^{(1)sd}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\psi^{(0)}_B(\mathbf{p}), \tag{21}$$

$$\chi_b(w) \cong \bar\Lambda\left(\frac{2}{w+1}\right)^{1/2}\int\frac{d^3p}{(2\pi)^3}\bar\psi^{(0)}_{D^{(*)\prime}}\left(\mathbf{p}+\frac{2\epsilon_q}{M_{D^{(*)\prime}}(w+1)}\mathbf{\Delta}\right)\left[\psi^{(1)si}_B(\mathbf{p})-3\psi^{(1)sd}_B(\mathbf{p})\right], \tag{22}$$

where $\mathbf{\Delta}^2 = M^2_{D^{(*)\prime}}(w^2-1)$. Here we used the expansion for the S-wave meson wave function

$$\psi_M = \psi^{(0)}_M + \bar\Lambda_M\varepsilon_Q\left(\psi^{(1)si}_M + d_M\psi^{(1)sd}_M\right) + \cdots,$$

where $\psi^{(0)}_M$ is the wave function in the limit $m_Q \to \infty$, $\psi^{(1)si}_M$ and $\psi^{(1)sd}_M$ are the spin-independent and spin-dependent first order $1/m_Q$ corrections, $d_P = -3$ for pseudoscalar and $d_V = 1$ for vector mesons. The symbol $\cong$ in the expressions (19)–(22) for the subleading functions $\tilde\chi_i(w)$ means that the corrections suppressed by an additional power of the ratio $(w-1)/(w+1)$, which is equal to zero at $w=1$ and less than 1/6 at $w_{\max}$, were neglected. Since the main contribution to the decay rate comes from the values of form factors close to $w=1$, these corrections turn out to be unimportant.

It is clear from the expression (17) that the leading order contribution vanishes at the point of zero recoil ($\mathbf{\Delta}=0, w=1$) of the final $D^{(*)\prime}$ meson, since the radial parts of the wave functions $\Psi_{D^{(*)\prime}}$ and Ψ_B are orthogonal in the infinitely heavy quark limit. The $1/m_Q$ corrections to the current also do not contribute at this kinematical point for the same reason. The only nonzero contributions at $w=1$ come from corrections to the Lagrangian [1] $\tilde\chi_1(w)$, $\tilde\chi_3(w)$ and $\chi_b(w)$. From Eqs. (16) one can find for the form factors contributing to the decay matrix elements at zero recoil

$$\begin{aligned} h_+(1) &= \varepsilon_c\left[2\tilde\chi_1(1)+12\tilde\chi_3(1)\right]+\varepsilon_b\chi_b(1),\\ h_{A_1}(1) &= \varepsilon_c\left[2\tilde\chi_1(1)-4\tilde\chi_3(1)\right]+\varepsilon_b\chi_b(1). \end{aligned} \tag{23}$$

Such nonvanishing contributions at zero recoil result from the first order $1/m_Q$ corrections to the wave functions (see Eq. (22) and the last terms in Eqs. (19), (21)). Since the kinematically allowed range for these decays is not broad ($1 \le w \le w_{\max} \approx 1.27$) the relative contribution to the decay rate of such small $1/m_Q$ corrections is substantially increased. Note that the terms $\varepsilon_Q(\bar\Lambda^{(n)}-\bar\Lambda)\xi^{(n)}(w)/(w-1)$ have the same behaviour near $w=1$ as the leading order contribution, in contrast to decays to the ground state $D^{(*)}$ mesons, where

[1] There are no normalization conditions for these corrections contrary to the decay to the ground state $D^{(*)}$ mesons, where the conservation of vector current requires their vanishing at zero recoil [26].

TABLE I: Decay rates Γ (in units of $|V_{cb}/0.04|^2 \times 10^{-15}$ GeV) and branching fractions BR (in %) for B decays to radially excited $D^{(*)\prime}$ mesons in the infinitely heavy quark limit and taking account of first order $1/m_Q$ corrections. $\Sigma(B \to D^{(*)\prime} e\nu)$ represent the sum over both channels. R' is a ratio of branching fractions taking account of $1/m_Q$ corrections to branching fractions in the infinitely heavy quark limit.

	$m_Q \to \infty$		With $1/m_Q$		
Decay	Γ	Br	Γ	Br	R'
$B \to D' e\nu$	0.53	0.12	0.92	0.22	1.74
$B \to D^{*\prime} e\nu$	0.70	0.17	0.78	0.18	1.11
$\Sigma(B \to D^{(*)\prime} e\nu)$	1.23	0.29	1.70	0.40	1.37

$1/m_Q$ corrections are suppressed with respect to the leading order contribution by the factor $(w-1)$ near this point (this result is known as Luke's theorem [26]). Since inclusion of first order heavy quark corrections to B decays to the ground state $D^{(*)}$ mesons results in approximately a 10-20% increase of decay rates [12, 18], one could expect that the influence of these corrections on decay rates to radially excited $D^{(*)\prime}$ mesons will be more essential. Our numerical analysis supports these observations.

We can now calculate the decay branching fractions by integrating double differential decay rates. Our results for decay rates both in the infinitely heavy quark limit and taking account of the first order $1/m_Q$ corrections as well as their ratio

$$R' = \frac{\mathrm{Br}(B \to D^{(*)\prime} e\nu)_{\mathrm{with}\, 1/m_Q}}{\mathrm{Br}(B \to D^{(*)\prime} e\nu)_{m_Q \to \infty}}$$

are presented in Table I. We find that both $1/m_Q$ corrections to decay rates arising from corrections to HQET Lagrangian (19)–(22), which do not vanish at zero recoil, and corrections to the current (18), vanishing at zero recoil, give significant contributions. In the case of $B \to D' e\nu$ decay both types of these corrections tend to increase the decay rate leading to approximately a 75% increase of the $B \to D' e\nu$ decay rate. On the other hand, these corrections give opposite contributions to the $B \to D^{*\prime} e\nu$ decay rate: the corrections to the current give a negative contribution, while corrections to the Lagrangian give a positive one of approximately the same value. This interplay of $1/m_Q$ corrections only slightly ($\approx 10\%$) increases the decay rate with respect to the infinitely heavy quark limit. As a result the branching fraction for $B \to D' e\nu$ decay exceeds the one for $B \to D^{*\prime} e\nu$ after inclusion of

first order $1/m_Q$ corrections. In the infinitely heavy quark mass limit we have for the ratio $Br(B \to D'e\nu)/Br(B \to D^{*\prime}e\nu) = 0.75$, while the account for $1/m_Q$ corrections results in the considerable increase of this ratio to 1.22.

In Table I we also present the sum of the branching fractions over first radially excited states. Inclusion of $1/m_Q$ corrections results in approximately a 40% increase of this sum. We see that our model predicts that 0.40% of B meson decays go to the first radially excited D meson states. If we add this value to our prediction for B decays to the first orbitally excited states 1.45% [27], we get the value of 1.85%. This result means that approximately 2% of B decays should go to higher charmed excitations.

IV. RARE RADIATIVE $B \to K_2^*(1430)\gamma$ DECAY

In the standard model $B \to K^{**}\gamma$ decays are governed by the contribution of the electromagnetic dipole operator O_7 to the effective Hamiltonian which is obtained by integrating out the top quark and W boson and using the Wilson expansion [8]:

$$O_7 = \frac{e}{16\pi^2}\bar{s}\sigma^{\mu\nu}(m_b P_R + m_s P_L)bF_{\mu\nu}, \qquad P_{R,L} = (1 \pm \gamma_5)/2. \tag{24}$$

The matrix elements of this operator between the initial B meson state and the final state of the orbitally excited tensor K_2^* meson have the following covariant decomposition

$$\begin{aligned}
\langle K_2^*(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}b|B(p)\rangle &= ig_+(k^2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu\beta}\frac{p_\beta}{M_B}k^\lambda(p+p')^\sigma,\\
\langle K_2^*(p',\epsilon)|\bar{s}ik_\nu\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle &= g_+(k^2)\left(\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B}(p+p')_\mu - \epsilon^*_{\mu\beta}\frac{p^\beta}{M_B}(p^2-p'^2)\right)\\
&\quad +g_-(k^2)\left(\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B}k_\mu - \epsilon^*_{\mu\beta}\frac{p^\beta}{M_B}k^2\right)\\
&\quad +h(k^2)((p^2-p'^2)k_\mu - (p+p')_\mu k^2)\epsilon^*_{\beta\gamma}\frac{p^\beta p^\gamma}{M_B^2 M_{K_2^*}},
\end{aligned} \tag{25}$$

where $\epsilon_{\mu\nu}$ is a polarization tensor of the final tensor meson and $k = p - p'$ is the four momentum of the emitted photon. The exclusive decay rate for the emission of a real photon ($k^2 = 0$) is determined by the single form factor $g_+(0)$ and is given by

$$\Gamma(B \to K_2^*\gamma) = \frac{\alpha}{256\pi^4}G_F^2 m_b^5|V_{tb}V_{ts}|^2|C_7(m_b)|^2 g_+^2(0)\frac{M_B^2}{M_{K_2^*}^2}\left(1-\frac{M_{K_2^*}^2}{M_B^2}\right)^5\left(1+\frac{M_{K_2^*}^2}{M_B^2}\right), \tag{26}$$

where V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements and $C_7(m_b)$ is the Wilson coefficient in front of the operator O_7. It is convenient to consider the ratio of exclusive to

inclusive branching fractions, for which we have

$$R_{K_2^*} \equiv \frac{BR(B \to K_2^*(1430)\gamma)}{BR(B \to X_s\gamma)} = \frac{1}{8} g_+^2(0) \frac{M_B^2}{M_{K_2^*}^2} \frac{\left(1 - M_{K_2^*}^2/M_B^2\right)^5 \left(1 + M_{K_2^*}^2/M_B^2\right)}{\left(1 - m_s^2/m_b^2\right)^3 \left(1 + m_s^2/m_b^2\right)}. \tag{27}$$

The recent experimental value for the inclusive decay branching fraction [28]

$$BR(B \to X_s\gamma) = (3.15 \pm 0.35 \pm 0.41) \times 10^{-4}$$

is in a good agreement with theoretical calculations.

For the calculation of the decay matrix elements in our model we use the same framework as in previous calculations of B to excited D decays. However, for a heavy-to-light transition we cannot expand $\Gamma^{(2)}$ contribution in inverse powers of the s quark mass. Instead we expand this contribution in inverse powers of the large recoil momentum $|\mathbf{\Delta}| \sim m_b/2$ of the final K_2^* meson. The resulting expressions for the form factor $g_+(0)$ up to the second order in $1/m_b$ can be found in [29].

We can check the consistency of our expressions for $g_+(0)$ by taking the formal limit of b and s quark masses going to infinity. [2] In this limit according to the heavy quark effective theory [30] the function $\xi_F = 2\sqrt{M_B M_{K_2^*}} g_+/(M_B + M_{K_2^*})$ should coincide with the Isgur-Wise function τ for the semileptonic B decay to the orbitally excited tensor D meson, $B \to D_2^* e\nu$. Such semileptonic decays have been considered by us in Ref. [27]. It is easy to verify that the equality of ξ_F and τ is satisfied in our model if we also use the expansion in $(w-1)/(w+1)$ (w is a scalar product of four-velocities of the initial and final mesons), which is small for the $B \to D_2^* e\nu$ decay [27]. Calculating the ratio of the form factor $g_+(0)$ in the infinitely heavy b and s quark limit to the same form factor in the leading order of expansions in inverse powers of the b quark mass and large recoil momentum $|\mathbf{\Delta}|$ we find this ratio to be equal to $M_B/\sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.965$. The corresponding ratio for the form factor $F_1(0)$ of the exclusive rare radiative B decay to the vector K^* meson [13] is equal to $M_B/\sqrt{M_B^2 + M_{K^*}^2} \approx 0.986$. Therefore we conclude that the form factor ratios $g_+(0)/F_1(0)$ in the leading order of these expansions differ by factor $\sqrt{M_B^2 + M_{K^*}^2}/\sqrt{M_B^2 + M_{K_2^*}^2} \approx 0.98$. This is the consequence of the relativistic dynamics leading to the effective expansion in inverse powers of the s quark energy $\epsilon_s(p+\Delta) = \sqrt{(\mathbf{p}+\mathbf{\Delta})^2 + m_s^2}$, which is large in one case due to the large s quark mass and in the other one due to the large recoil momentum

[2] As it was noted above such limit is justified only for the b quark.

TABLE II: Our results in comparison with other theoretical predictions and experimental data for branching fractions and their ratios $R_{K^*} \equiv \frac{BR(B\to K^*\gamma)}{BR(B\to X_s\gamma)}$, $R_{K_2^*} \equiv \frac{BR(B\to K_2^*\gamma)}{BR(B\to X_s\gamma)}$, $r \equiv \frac{BR(B\to K_2^*\gamma)}{BR(B\to K^*\gamma)}$. Our values for the $B \to K^*\gamma$ decay are taken from Ref. [13].

Value	our	Ref. [9]	Ref. [10]	Ref. [11]	Exp. [7]
$BR(B \to K^*\gamma) \times 10^5$	4.5 ± 1.5	1.35	$1.4 - 4.9$	4.71 ± 1.79	$4.55^{+0.72}_{-0.68} \pm 0.34^a$
					$3.76^{+0.89}_{-0.83} \pm 0.28^b$
R_{K^*} (%)	15 ± 3	4.5	$3.5 - 12.2$	16.8 ± 6.4	
$BR(B \to K_2^*\gamma) \times 10^5$	1.7 ± 0.6	1.8	$6.9 - 14.8$	1.73 ± 0.80	$1.66^{+0.59}_{-0.53} \pm 0.13$
$R_{K_2^*}$ (%)	5.7 ± 1.2	6.0	$17.3 - 37.1$	6.2 ± 2.9	
r	0.38 ± 0.08	1.3	$3.0 - 4.9$	0.37 ± 0.10	$0.39^{+0.15}_{-0.13}$

$\mathbf{\Delta}$. As a result both expansions give similar final expressions in the leading order. Thus we can expect that the ratio r from (2) in our calculations should be close to the one found in the infinitely heavy s quark limit [11].

The results of numerical calculations are given in Table II. There we also show our previous predictions for the $B \to K^*\gamma$ decay [13]. Our results are confronted with other theoretical calculations [9, 10, 11] and recent experimental data [7]. We find a good agreement of our predictions for decay rates with the experiment and estimates of Ref. [11]. Other theoretical calculations substantially disagree with data either for $B \to K^*\gamma$ [9] or for $B \to K_2^*\gamma$ [10] decay rates. As a result our predictions and those of Ref. [11] for the ratio r from (2) are well consistent with experiment, while the r estimates of [9] and [10] are several times larger than the experimental value (see Table II). As it was argued above, it is not accidental that r values in our and Ref. [11] approaches are close. The agreement of both predictions for branching fractions could be explained by some specific cancellation of finite s quark mass effects and relativistic corrections which were neglected in Ref. [11]. We believe that our analysis is more consistent and reliable. We do not use the ill-defined limit $m_s \to \infty$, and our quark model consistently takes into account some important relativistic effects, for example, the Lorentz transformation of the wave function of the final K_2^* meson (see Eq. (14)). Such a transformation turns out to be very important, especially for B decays to orbitally excited mesons [27]. The large value of the recoil momentum $|\mathbf{\Delta}| \sim m_b/2$ makes relativistic effects to play a significant role. On the other hand this fact simplifies

our analysis since it allows to make an expansion both in inverse powers of the large b quark mass and in the large recoil momentum.

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